Suska Journal of Mathematics Education
(p-ISSN: 2477-4758|e-ISSN: 2540-9670)
Vol. 6, No. 1, 2020, hal. 009-016

Students' Mathematical Reasoning in Learning of Transformation Geometry

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Abstrak. Penalaran matematis adalah komponen yang harus dibangun oleh siswa. Penalaran matematis berperan penting dalam menyelesaikan masalah dan menyampaikan ide dalam belajar matematika. Siswa belajar bahwa penalaran tidak hanya sekedar bagaimana meyakinkan diri sendiri dan teman tetapi juga harus meyakinkan ke komunitas yang lebih luas dari suatu kelas. Lebih jauh, penalaran dapat menjadi kendaraan untuk membentuk pengetahuan matematika yang dapat dikembangkan, dan diakses oleh anggota dari komunitas dalam kelas. Materi transformasi geometri merupakan salah satu materi yang sangat dekat dalam kehidupan sehari-hari. Penelitian ini adalah penelitian deskriptif kualitatif yang bertujuan untuk mendiskripsikan penalaran matematis siswa dalam mempelajari materi geometri transformasi. Subjek dalam penelitian ini adalah siswa kelas XI MIPA 5 di MAN Kota Batu, Malang, Jawa Timur. Melalui proses penalaran matematis siswa mampu memenuhi kompetensi dasar yaitu menganalisis dan membandingkan transformasi dan komposisi transformasi dengan menggunakan matriks.**ABSTRAK.** Gunakan huruf Garamond 10pt dengan satu spasi antar baris. Left indent 2.5cm dan right indent 0 cm dan panjang maksimum terdiri dari 250 kata. Abstrak minimal harus memuat tujuan utama, metodologi, temuan utama dan simpulan.

Kata Kunci: penalaran matematis, transformasi geometri, pencerminan

Abstract. Mathematical reasoning is component that must be built by students. Mathematical reasoning plays an important role in solving problems and delivering ideas in learning mathematics. Students learn that reasoning is not just how to convince themselves and friends but also to convince the wider community of a class. Furthermore, reasoning can be a vehicle for shaping mathematical knowledge that can be developed, and accessed by members of the community in the class. Geometry transformation material is one material that is very close to everyday life. This research is a qualitative descriptive study that aims to describe the mathematical reasoning of students in learning the transformation geometry. The subjects in this study were students of class XI MIPA 5 in MAN Batu, Malang, East Java. Through the process of mathematical reasoning, students are able to fulfill basic competencies, namely analyzing and comparing transformation and composition of transformations using matrix.

Keywords: mathematical reasoning, geometry transformation, reflection.

INTRODUCTION

Mathematics is a scientific discipline that has an important role in human life. The development of technology in formation and communication is now inseparable from the intervention of mathematics, so the importance of mathematics learning is given at every level of education. Regarding the importance of mathematics in human life, it is not balanced with good mathematical learning outcomes. Not infrequently mathematics is considered a scourge for

students and is considered difficult. These difficulties are closely related to the process of solving problems, especially in mathematical reasoning.

Mathematical reasoning is component that must be built by students. Mathematical reasoning plays an important role in solving problems and delivering ideas in learning mathematics (Ayal & Kusuma, 2016). A reasoning aspect should be an important aspect of mathematics learning. The reasoning and arguing abilities are very important for many different domains and have a special role in mathematics (Wong, 2016).

Research on mathematical reasoning has been done by many researchers (e.g., Paliwal & Baroody, 2020; McFeetors & Palfy, 2018; O'neill, 2019; Walkington et al., 2019; Norqvist et al., 2019). Paliwal & Baroody (2020) researched about the addition reasoning strategy to solve the problem of substraction. McFeetors & Palfy (2018) in his research found that students' mathematical reasoning is increasing by giving board games, while O'neill (2019) Associates mathematical reasoning with artificial intelligence. Walkington et.al. (2019) Examining the influence of a hand movement in the mathematical reasoning ability. In his research Walkington et.al. (2019) found that the barriers to hand movements did not affect the mathematical reasoning.

Mathematical reasoning is a brain habit that when it is developed properly and consistently, it will facilitate communication of mathematics both in writing and verbally. Delivering mathematical opinions and ideas is not an easy thing because it requires careful and good reasoning. Students learn that reasoning is not just how to convince themselves and friends but also to convince the wider community of a class. Furthermore, reasoning can be a vehicle for shaping mathematical knowledge that can be developed, and accessed by members of the community in the classroom (Bieda, 2010). Mathematics can be the main role in the development of personal authority and reasoning that convinces oneself and others as a guarantee (Mason, 2014).

Hollebrands in Febrian & Perdana (2017) argue that transformation geometry learning can provide opportunities for students to think about other important mathematical concepts such as symmetry, congruence, function, etc. and realize that geometry transformation involves various disciplines and allows for reasoning or developing reasoning. Transformation geometry reasoning is the process of thinking, understanding, and making decisions based on logical processes related to the problem of geometry transformation. Reasoning in geometry transformation can be in the form of graphical / visual reasoning and algebraic reasoning in accordance with the nature of geometric transformation which can be approached by two methods: graphic and algebraic (Mashingaidze, 2017). Geometry transformation reasoning is a strong capital to be able to solve the problem of transformation geometry.

According to NCTM (2000: 58) there are four Standards of reasoning and proof, including: 1) recognizing reasoning and proof as the basic aspect of mathematics; 2) making and investigating mathematical allegations; 3) developing and evaluating mathematical arguments and verification; 4) selecting and using various types of reasoning and proof methods.

In the revised 2013 curriculum in 2017, geometry transformation material is delivered in the odd semester of class XI. Of the four core competencies, the writer emphasizes the third core competency, namely, understanding, applying, and analyzing factual knowledge, conceptual, procedural, and metacognitive knowledge based on the curiosity about science, technology, art, culture, and humanity with insight into humanity, nationality, and civilization regarding the causes of the phenomenon of events, as well as applying procedural knowledge to the specific field of study according to the talents and interests to solve problems. Of all the basic competencies that exist, this discussion will present the basic competencies of 3.5 to analyze and compare transformations and composition of transformations using matrix.

Geometry transformation consists of translation, reflection, rotation, and dilatation. Of the four types of transformations in this discussion, the focus will be on the reflection sub-section. In the students book of class XI Kemendikbud (2017), reflection is explained as follows: 1) Reflection of point O (0,0); 2) Reflection of axis X; 3) Reflection of axis Y; 4) Reflection of line y = x; 5); Reflection of line x = h; 6) Reflection of line y = k; 7) Reflection of line y = -x.

From the description above, the researcher will conduct research on reasoning in geometry transformation learning. Through the process of mathematical reasoning, students are expected to be able to fulfill basic competencies, namely analyzing and comparing transformations and composition of transformations using matrix.

METHODS

In this study, the method used was qualitative descriptive. The researcher designed learning in geometry transformation material which focused on the standard of reasoning and proof process. The reasoning standard used 4 standards by NCTM, namely: 1) recognizing reasoning and proof as the basic aspect of mathematics; 2) making and investigating mathematical allegations; 3) developing and evaluating mathematical arguments and verification; 4) selecting and using various types of reasoning and proof methods. In this study, focused only on mathematical reasoning.

After finishing the design, the learning design was implemented in class XI MIPA 5 MAN Batu. Students were given problems about a quadrilateral reflected by line. Then students were asked to reflect using the obtained matrix equation. After that the students verified through definition whether the results of the reflection were the same if the points were searched using the definition of reflection. The design and results of learning were analyzed and described qualitatively.

RESULTS AND DISCUSSION

Before conducting the learning, the teacher had to prepare a learning design in accordance with the objectives to be achieved. In this learning design, the activities carried out by the teacher were focused on the standard process of reasoning. In addition to designing learning designs, the teacher also made worksheets that could support the running of the standard process of reasoning correctly. There are four standard of proof and reasoning processes according to NCTM (2000), namely: 1) recognizing reasoning as the basic aspect of mathematics; 2) making and investigating mathematical allegations; 3) developing and evaluating mathematical arguments and verification; 4) selecting and using different types of reasoning methods.

In the initial activity, the teacher reminded of geometry transformation material especially reflections that had been received by students at the junior high school level. From the algebraic understanding or equation that had been accepted by students, it would be associated with matrix equation. The teacher carried out the first standard of reasoning process by asking students to do geometrical reasoning for the images provided. From geometrical reasoning, the characteristics or algebraic equation of each reflection would be gotten.

After students got the algebraic equation for each reflection, students were asked to associate the algebraic equation with the concept of matrix equation. This step was the second standard process. Then the third standard process would be achieved when students checked between the matrix equation obtained and algebraic equation that had been studied in junior high

school. Then in the last step students were asked to solve the problem given using the algebraic equation found, then associated with the definition of reflection.

The following is the result of reflection material learning on geometry transformation using four standard processes of reasoning . In the first part (part A) students were asked to observe the images that had been presented. Then the students mentioned the events that happened to the pictures.

In part B, students observed the reflection of axis given in the picture, then students did geometrical reasoning and formulated the definition of reflection in accordance with the results of geometrical reasoning. In formulating definitions, students not only observed the picture but also related the facts to the picture with the reflection characteristics they had learned at the junior high school level. From the results of the observation, students would easily formulate the definition of reflection using their own language.

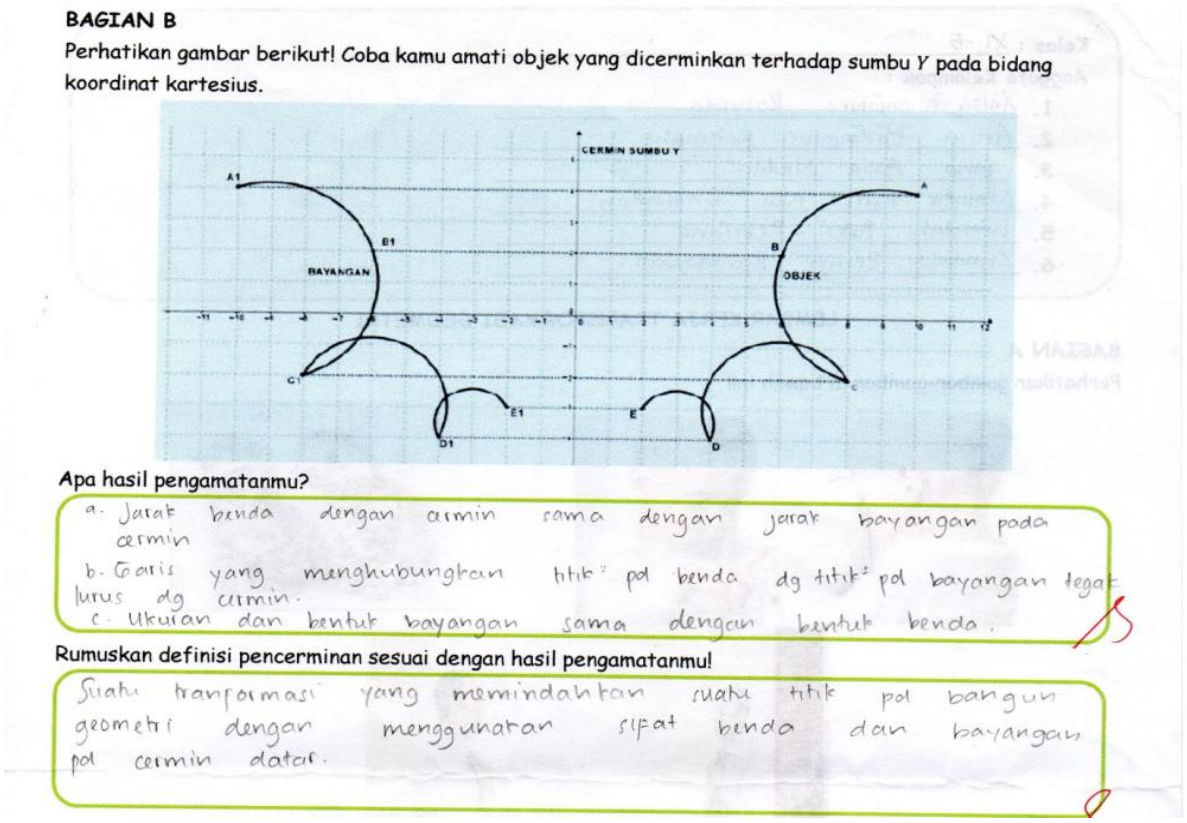

Figure 1 - Part B of Student's Worksheet

In parts A and B the students carried out the first standard of reasoning processes, namely recognizing reasoning as the basic aspect of mathematics. It was expected that after students worked on parts A and B, students would realize that to formulate a definition of a given fact it was necessary to have a reasoning process. In this case the reasoning carried out by students was geometry reasoning because students got conclusions according to the geometry characteristics of the problem given. It can be seen from figure 1 that a student did geometry reasoning graphically, according to what was revealed by Maria, Aiso, & Alexander (2008) that understanding a geometric concept can be done by reasoning with graphics.

After the students defined reflection in their own language, in the next section students would verify their arguments and formulate a definition of reflection in the language of mathematics. In part C students were asked to observe several geometric objects which were reflected in several flat mirrors, namely point O (0,0), axis X, axis Y, line x = h, line y = k, line y = x, and line y = -x. Students' arguments could be in the form of sentences which were then written algebraically. Then from the algebraic equation, students formulated the matrix equation.sults and discussion section presents the results of research in the form of tables, charts, diagrams, or narratives were then given meaning or description of any research results are presented. It also presents the discussion contains a description of the researchers on the study results either as expected or not as expected.

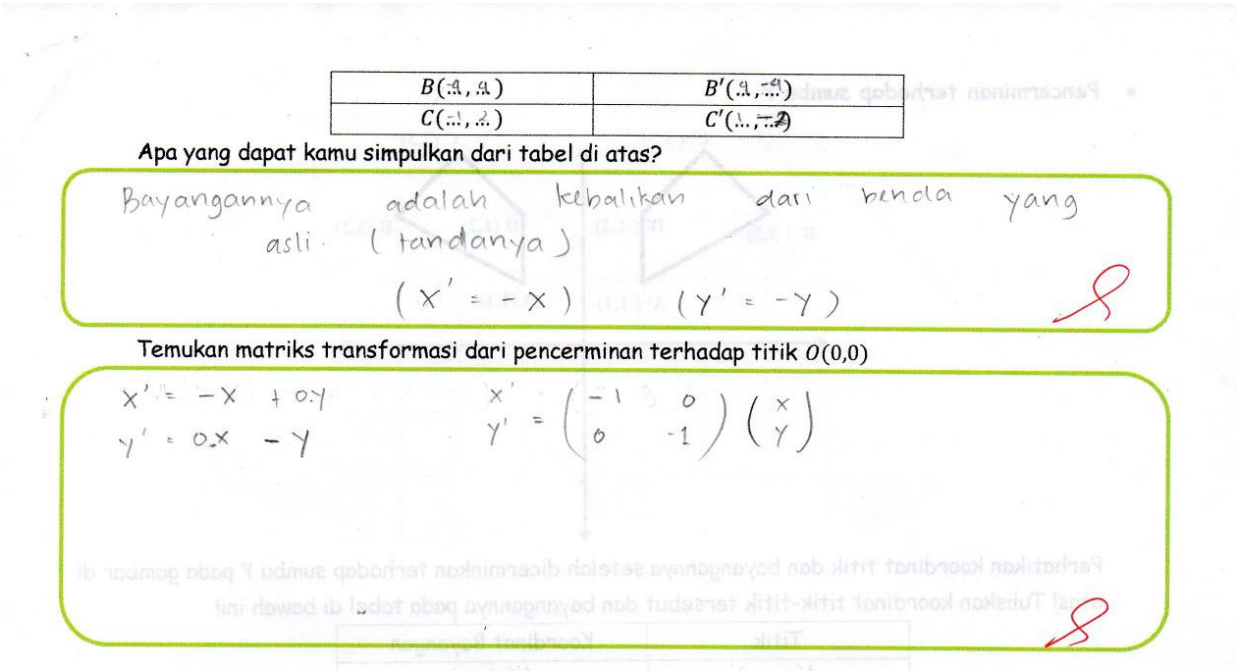

Figure 2 - Part C of Student's Worksheet

Part C was an implementation of the second and third standard processes, namely making and investigating mathematical allegations and developing and evaluating mathematical arguments and verification. In the first column, the students used the argument in the form of sentences, which were then modeled in mathematical form. Furthermore, from the algebraic equation that had been obtained, students associated with the concept of matrix equations until the conclusion of the definition of reflection was obtained in the form of a matrix equation. It was in accordance with what was conveyed by Mashingaidze (2017) that in addition to graphic or image forms, geometry reasoning can also be realized in the form of algebra.

Then students worked on the exercise in sections D and E to apply what they had obtained in the previous sections. Part D exercise was a matching exercise, while part E was an objective or multiple choice exercise. At the end, namely part F, students were asked to solve the problem given by using the matrix equation found, then associated with the definition of reflection geometrically. In this section, students carried out the fourth standard process, namely selecting and using various types of reasoning methods.

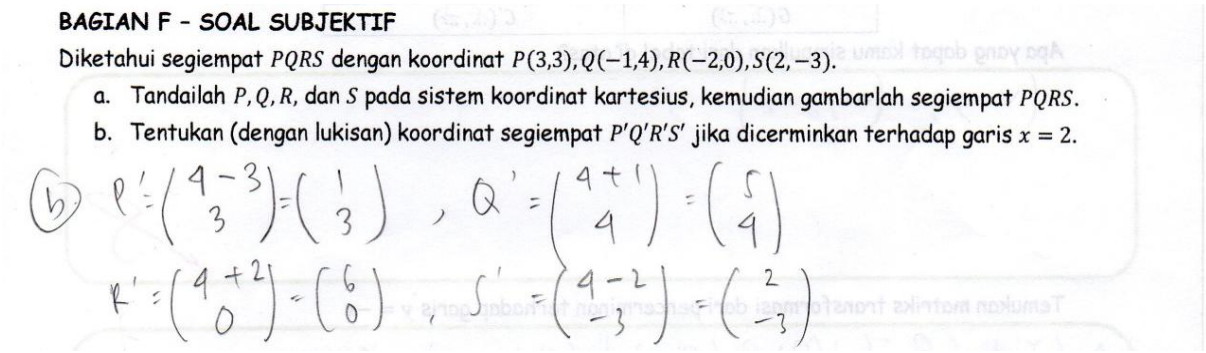

Figure 3 - Part F of Student's Worksheet

In Figure 3, students did reasoning algebraically to find the shadow of points reflected in line $x = 2$. However, there were also students who used graphics or images to determine the result of reflection. Figure 4 below shows the results of students' works in determining shadow of points using graphic or images reasoning.

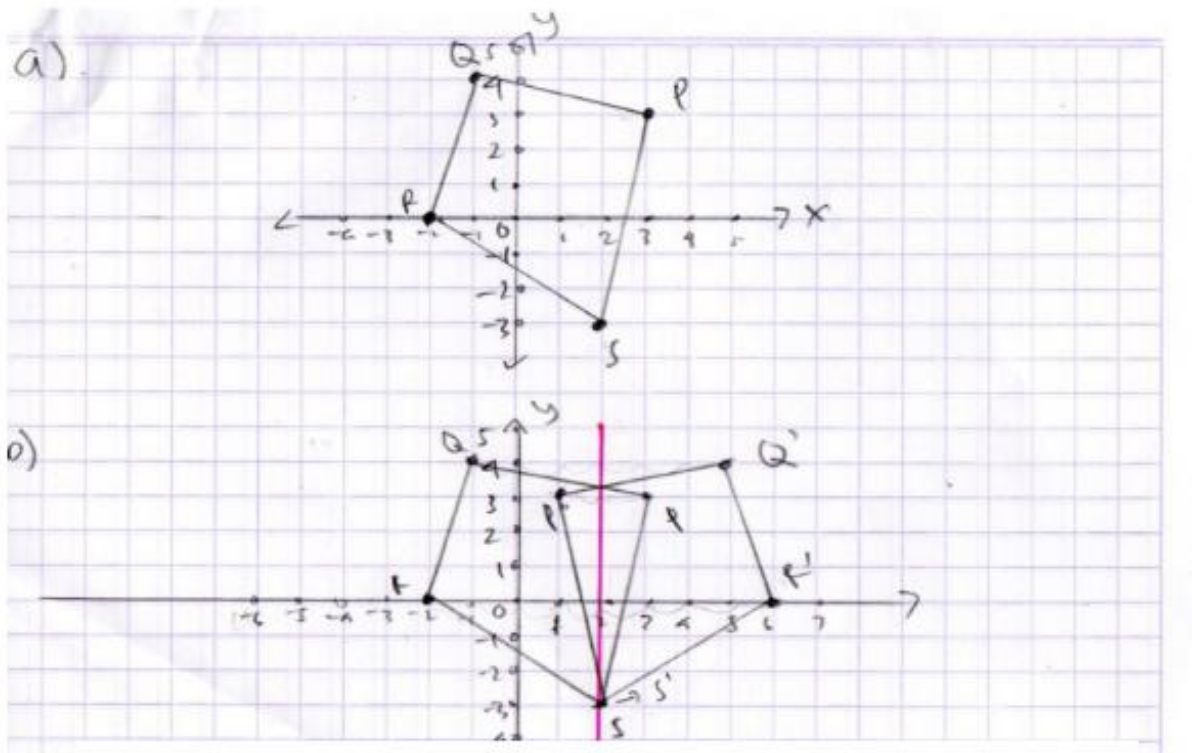

Figure 4 - Other Answers for Part F of Student's Worksheets

From the results described above, it can be seen that understanding the concept of geometry transformation requires a standard of reasoning process. If students have implemented the standard of reasoning in learning, students do not need to memorize the answer pattern anymore. Students can also choose what geometry reasoning they will use to solve the problem given.

CONCLUSION

Mathematical reasoning plays an important role in solving problems and delivering ideas in learning mathematics. Most students believe in empirical evidence but are unsure of the evidence in the form of deductive arguments. More specifically, some students feel that verification must be provided by the teacher and students have little desire to do verification (Hong, Choi, Reasoning, Hong, & Choi, 2018). This certainly will cause student's reasoning ability to be low. Therefore in each curriculum the standard processes of reasoning need to be applied to improve students' reasoning ability (Otten, Gilbertson, & Males, 2014). In addition, carrying out reasoning will make students not only memorize the answer procedure, but also fully understand the concept of what they are learning (Liu & Manouchehri, 2013).

ACKNOWLEDGEMENT

Thank you to the students and teachers at MAN Batu, Malang, who have helped in the implementation of this research.

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